Subjective And Objective Risk Tolerance: Implications For Optimal Portfolios

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The distinction between subjective and objective risk tolerance is illustrated by expected utility analyses of portfolios. Optimal portfolios were derived for one, 5, and 20 year investment horizons for 6 major financial asset categories. The important aspects of objective risk tolerance are the proportion of an investor's total wealth (including human wealth) in financial assets, and the investment horizon. Even investors with very low subjective risk tolerance levels should have aggressive portfolios if their horizons are 20 years or more.

Key Words: *Risk tolerance, Portfolios, Investment*

In investing for long term goals, the allocation of asset categories in the portfolio is one of the most crucial decisions. Most people are not willing to take above average risks to obtain above average returns on their investments (Avery & Elliehausen, 1986). It is likely that many investors focus too much on short term volatility, especially for investing for retirement.

Many financial planners stress the concept of the client's risk tolerance, and give questionnaires to assess risk tolerance. Malkiel (1996, p. 401) stated that "The risks you can afford to take depend on your total financial situation, including the types and sources of your income exclusive of investment income." This article proposes that risk tolerance be into two parts: subjective risk tolerance based on the economic concept of risk aversion, and objective risk tolerance, based on Malkiel's idea of the objective financial situation of the household, including the investment horizon for each goal. The expected utility of alternative portfolios is calculated. Optimal portfolios for various combinations of objective and subjective risk tolerance are identified.

Risk Versus Return

It is well known that stocks have a higher mean rate of return than bonds.[a] Between the beginning of 1926 and the end of 1997, after adjusting for inflation, a dollar invested in small stocks would have grown to $613, compared to $203 for large stocks, $6 for corporate bonds, $4 for government intermediate and long bonds, and under $2 for Treasury bills (Ibbotson Associates, 1998). If the long run patterns from the past are the best indicators of the future, an investor who wanted to maximize expected return and had a long term perspective would have a portfolio consisting only of small stocks. In order to obtain higher rates of return, however, the investor must accept greater risk, or at least greater volatility. However, even this supposed truism is not true in the long run. Small stocks performed best of six investment categories in 48 out of 51 possible consecutive 20 year periods between 1926 and 1995, and large stocks performed best in the other three 20 consecutive year periods (Ibbotson Associates, 1996, p. 43).

If all future 20 consecutive year periods resemble these 51 time periods, small stocks present the least risk to the investor. A 20 year investment horizon may not be appropriate for many investors, however. The standard deviations of one year returns of the Ibbotson investment categories range from 34% for small stocks to 3% for Treasury bills (Ibbotson Associates, 1996, p. 33). How should an investor balance the mean return and the volatility as represented by the standard deviations according to his/her own situation?

Samuelson (1969) listed several reasons commonly given why " ... a young businessman can take more risk in the financial market than an old widow..." : 1) The "businessman" is more affluent than the widow; 2) expects higher earnings in the future; 3) can "recoup" any current losses in the future; 4) has a much longer investment horizon. These explanations can be viewed as an application of the life cycle model offered by Ando

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and Modigliani (1963). Similarly, Malkiel (1990, p. 339) suggests:

> Investment strategy must be keyed to a life cycle. It is simple common sense to say that a thirty-four-year-old and a sixty-four-year old saving for retirement may prudently use different financial instruments to accomplish their goals. A thirty-four-year-old -- just beginning to enter the peak years of income earnings -- can use wages to cover any losses from increased risk ..."

Malkiel (1990) suggests that the portion of the portfolio for stocks should decrease as a person ages. His suggested stock share goes from 70% for a 25 year old to 30% for a 70 year old. The pattern is similar to the advice that the percentage in fixed income investments should equal your age (Willette, 1995). Malkiel (1990) also suggests that the individual's attitude toward risk should be considered, although he is vague on exactly how risk tolerance should be incorporated into construction of a portfolio.

Delaney and Reichenstein (1996) and Reichenstein and Delaney (1995) offer a similar argument. They analyzed household portfolio composition in a broad picture which included real estate and human wealth. They suggest that human wealth is usually the dominant asset for young and mid-aged households, which means financial assets would be a tiny fraction of their wealth. It is quite reasonable for a young or mid-aged family to hold an all-stock investment portfolio, because they could easily offset any disastrous returns in the short run through adjusting future consumption and savings.

In contrast to the Malkiel (1990) and Delaney and Reichenstein (1996) recommendations, the portfolios recommended by 13 brokerage houses in 1994 had stocks ranging from 48% to 65% of the portfolio, with an average of 52% for stocks (Dorfman, 1995, p. C1).

Shortfall Analyses

In considering risk versus return, various approaches have been taken, including a focus on the possibility of a shortfall in consumption or in some arbitrary goal (e.g., Leibowitz & Langetieg, 1989; Leibowitz & Kogelman, 1991; Ho, Milevsky & Robinson, 1994). Hanna and Chen (1996) and Chen and Hanna (1996) determined efficient portfolios for saving for intermediate term goals such as a college fund, and long term goals, such as retirement. For periodic investing for 20 years or more, the most aggressive portfolios were safer, in terms of having the highest worst case real accumulation, based on historical returns since 1926. However, for shorter investment horizons, an investor would have to decide on how much risk to accept, in effect a shortfall evaluation. An alternative method of evaluating such tradeoffs is use of an expected utility analysis.

Expected Utility

The most powerful *normative* model for decision making with uncertainty is the expected utility model (Schoemaker, 1982). There are many expositions of the model (Arrow, 1971; Deaton & Muellbauer, 1980; Machina, 1987), and the graphical illustration of risk averse versus risk seeking consumers is familiar (Deaton & Muellbauer, 1980). The apparent paradox posed by Friedman and Savage (1948), that many consumers both buy insurance and gamble, is more commonly explained today in terms of direct utility expected from the process of gambling. It is probably reasonable to assume that most consumers are risk averse (Bailey, Olson & Wonnacott, 1980). A rich set of results about optimal behavior under uncertainty can be derived by assuming that risk averse investors should maximize expected utility, with utility a function of wealth.

Expected utility maximization is a widely used approach for analyzing the optimality of portfolios. The mean-variance model developed by Markowitz (1952) was based on the concept of expected utility maximization. Many previous studies on optimal portfolios were trying to solve problems encountered through operationalizing the model in the real world or to loosen some of the strict assumptions. For example, Black (1972) expanded the mean-variance model by introducing the zero-beta portfolio; Alexander (1978) studied optimal portfolios with restricted borrowing; and Fama (1965) analyzed optimal portfolios under a general stable distribution instead of the normal distribution.

Another normative approach to implement the expected utility idea in finding an optimal portfolio is using direct utility maximization of utility functions and historical records of returns. This approach does not require strict assumptions concerning the distribution of returns imposed by the mean-variance model. Levy and Markowitz (1979) and Kroll, Levy and Markowitz (1984) showed both theoretically and empirically that the mean-variance method and the expected utility approach have similar optimal portfolios. The expected utility simulation method used in Kroll, Levy and Markowitz (1984) is almost identical with the one used in this paper, i.e., the optimal portfolio is defined as one that maximizes the expected utility among all feasible portfolios. In this article, diversified financial asset categories are used instead of the individual stocks used in Kroll, Levy and Markowitz's work. The expected utility approach has also been used to analyze alternative portfolio strategies (e.g., Knight & Mandell, 1995).

Relative Risk Aversion Utility functions can be characterized in terms of relative risk aversion, which is "... a measure of the concavity of the utility function or the disutility of consumption fluctuations" (Grossman & Shiller, 1981, p. 224). The higher the relative risk aversion, the more rapidly marginal utility decreases as consumption or wealth increases. One type of utility function used for analysis of investment decisions is the constant relative risk aversion utility function (e.g., Samuelson, 1990), which can be specified as shown in Equations 1 and 2.

$$U(W)=\frac{W^{1-x}}{1-x} \quad for\ \ x\neq 1 \tag{1}$$

$$U(W)=\ln(W) \quad for\ \ x=1 \tag{2}$$

where

x = relative risk aversion level

W = total wealth

There have been some attempts to estimate risk aversion by analyzing household portfolios and other decisions under risk (e.g., Friend & Blume, 1975a; Friend & Blume, 1975b). However, it is necessary to assume that households behave with full information and rationality in order for such empirical analysis to reveal risk aversion. An alternative approach is to use introspection. Kimball (1988) presented an intuitive way of evaluating one's own level or relative risk aversion. A modified version of Kimball's (1988) example developed by Hanna (1988) and cited by Fan, Chang and Hanna (1993, p. 50) was used to illustrate the concept of relative risk aversion. Risk tolerance can be operationalized as the inverse of relative risk aversion.

In the context of the expected utility model, relative risk aversion relates to the extra utility of increased consumption if the gamble pays off compared to the lost utility because of decreased utility if you lose the gamble. Risk neutrality (relative risk aversion level of zero, or infinite risk tolerance) implies accepting a 50% chance of death in exchange for a 50% chance of doubling your wealth, as wealth includes all possible resources for consumption. Therefore, risk neutrality is not plausible.[b]

Empirical Patterns of Risk Tolerance

There are few empirical studies that correspond *exactly* with the economic concept of risk aversion. The 1992 Survey of Consumer Finances asked respondents about how much risk they would take in making investments. Only 18% of all respondents under the age of 70 and not retired were willing take above-average or substantial risks to earn above-average or substantial returns, and 40% were not willing to take any risks (Sung & Hanna, 1996). One problem with this measure is that it does not necessarily reveal pure preferences, as an answer may depend upon the respondent's situation. For instance, someone who has no financial assets other than a checking account which is depleted each month may not be in a position to take any risk. The same person 10 years later may be in a position to invest retirement funds in stocks. Another problem with the measure of risk tolerance is that many people may confuse the volatility of higher return investments with a risk of longer term losses.

Barsky, Juster, Kimball, and Shapiro (1997) presented an experimental measure based on presenting a set of hypothetical questions to a large national sample of males aged 51 to 61. Their results imply that 35% of the respondents had a relative risk aversion level under 3.8, and 65% had a higher level. Relative risk aversion levels above 10 imply reluctance to accept a chance of a 5% drop in wealth in exchange for an equal chance of doubling wealth.[c]

Methods

Objective and Subjective Risk Tolerance

The effect of objective risk tolerance is investigated based on the investment horizon and the ratio of the household's financial assets to total wealth. The effect of subjective risk tolerance is investigated based on the investor's relative risk aversion. Low subjective risk tolerance is considered equivalent to relative risk aversion levels of 10 or more (Kimball, 1988; Hanna, 1988; Barsky, et al., 1997). Moderate subjective risk tolerance is considered equivalent to relative risk aversion levels of 3 to 9. High subjective risk tolerance is considered equivalent to relative risk aversion levels under 3.

Real Rates of Return

The real rate of return is the appropriate basis for evaluating investments. Tax considerations may make the nominal rate of return relevant. However, in this paper, tax considerations are ignored. This may be a reasonable assumption if the portfolio is tax sheltered. The results of this analysis may not be valid for retirees and those with portfolios too large to shelter completely. Nominal rates of return and the inflation rates were obtained from Ibbotson Associates (1996) for six categories of financial assets. The real rate of return was calculated by the authors.

Investment Horizon

The choice of investment horizon for analysis is of fundamental importance to the analysis of optimal portfolios. The investment horizon varies according to different investment goals and different investors. A 20 year investment horizon is appropriate for young household saving for retirement, however, it is clearly not valid for someone who is near retirement or whose current consumption depends on the portfolio. In this analysis, three investment horizons were used to find the optimal portfolio for short (one year), intermediate (5 year), and long (20 year) investment horizons. A one year analysis is appropriate for investors whose current consumption depends on the portfolio; the 5 year investment horizon is suitable for investors with intermediate goals, for example, saving for a down payment for a home; and a 20 year investment horizon for those saving for retirement when it is 20 or more years away. The Ibbotson (1996, p. 43) results for nominal returns give some idea of the well-know issue of risk versus return: for the 70 calendar year periods between 1926 and 1995, small stocks were best 31 times, although the worst year produced a 58% loss. For the 51 rolling 20 year periods, small stocks were best 48 times, with the worst annualized rate of return being 3%.

Wealth

The utility function approach is based on the idea that consumption is based on wealth. A consumer's non-investment income can also provide for consumption. Therefore, wealth should include not only net worth but also a measure of human wealth, which is defined as the present value of non-investment income. For the analyses presented in this article, human wealth and any other non-financial wealth are considered as safe assets, or at least uncorrelated with financial assets. This assumption is clearly not valid for many households, and the implications of the assumption are discussed in the conclusions section.

The proportion of financial assets to total wealth (including human wealth) had a median level of only 1% for all households in the United States, and the median level was only 7% only for households headed by someone aged 65 or older (Lee & Hanna, 1995). Only 10% of households had a ratio of financial assets to total wealth of 17% or more (Lee & Hanna, 1995). Therefore, for over 90% of households, a 20% loss in the value of all financial assets would represent less than a 1% loss in total wealth. For many households, as financial assets increase and human wealth decreases, the financial asset to wealth ratio will increase with age.[d]

Calculating Expected Utility of Portfolios

It was assumed that the asset return patterns of each possible period from 1926 through the end of 1995 (e.g., 66 overlapping 5 year periods) would be equally likely to occur in the future (Ibbotson Associates, 1996, p. 25). Many studies (e.g., Leibowitz & Krasker, 1988) have assumed a particular probability distribution for annual asset returns, such as normal or lognormal, then have randomly drawn returns to project long run distributions. This method requires that there is no cross-sectional correlation between asset classes and no serial correlation within an asset class. However, empirical studies have shown that cross sectional correlations and serial correlation do exist among the returns of investment instruments, such as stocks, bonds, and bills. In contrast, all that is assumed with the nonparametric projection method used in this article is that all future periods resemble periods from the beginning of 1926 to the end of 1995.[e] The probability of each set of rates of return is 1/66 (0.0152.) A simulation program was developed based on this assumption, with relative risk aversion set to range from 0 to 10, and financial assets as a proportion of total wealth set to range from 10% to 100%. The expected utility of all possible portfolios (in increments of 1% for each asset category) was calculated, and the portfolio with the highest expected utility for a particular combination of relative risk aversion and level of financial assets as a percent of total wealth was recorded as the optimal portfolio.

Results

20 Year Investment Horizon

For a 20 year investment horizon, a 100% small stock portfolio (i.e., an index fund matching the Ibbotson definition) dominates any other portfolio for every financial asset level and relative risk aversion level. A portfolio consisting of small stocks has higher expected utility than any other portfolio combination. Based on the assumptions used in this study, large company stocks, corporate bonds, intermediate government bonds and Treasury bills should not be included in the portfolio, if

Table 1
Optimal Portfolios for Selected Levels of the Ratio of Financial Assets to Total Wealth, for Relative Risk Aversion Level of 6, for One Year and 5 Year Time Frames.

		One Year Investment Horizon				Five Year Investment Horizon			
Relative Risk Aversion	Financial Assets as a Percentage of Total Wealth	Large Stocks	Small Stocks	Long-term Corporate Bonds	Intermediate Government Bonds	Large Stocks	Small Stocks	Long-term Corporate Bonds	Intermediate Government Bonds
2	10%	0%	100%	0%	0%	0%	100%	0%	0%
6	10%	0%	100%	0%	0%	0%	100%	0%	0%
10	10%	0%	100%	0%	0%	29%	71%	0%	0%
2	50%	0%	100%	0%	0%	16%	84%	0%	0%
6	50%	18%	32%	0%	50%	24%	35%	41%	0%
10	50%	10%	21%	0%	69%	17%	26%	0%	57%

A relative risk aversion level of 2 might correspond to high subjective risk tolerance. A level of 6 might correspond to moderate subjective risk tolerance, and a level of 10 might correspond to low subjective risk tolerance.

the investment horizon is 20 years or more.

Five and One Year Investment Horizons
Table 1 shows selected results for 5 and one year investment horizons, assuming relative risk aversion levels of 2, 6, and 10. Of the six Ibbotson financial assets, long-term government bonds and Treasury bills are never in an optimal portfolio. For households with financial assets comprising 10% of total wealth, optimal portfolios include only stocks, both for one and for 5 year investment horizons. For households with a relative risk aversion of 6 and financial assets representing 50% of total wealth, the patterns for the two investment horizons are similar, with the one year investment horizon seemingly more conservative than the corresponding 5 year horizon. Large plus small stocks comprise 50% of the optimal portfolio for the one year horizon and 59% of the optimal portfolio for the 5 year horizon. For the one year horizon, intermediate government bonds comprise 50% of the optimal portfolio, and for the 5 year horizon, corporate bonds comprise 41% of the optimal portfolio. There is a similar anomaly for a relative risk aversion level of 10 with financial assets representing 50% of wealth. The anomaly may reflect a defect in the method presented in this article for holding periods under 5 years.[f]

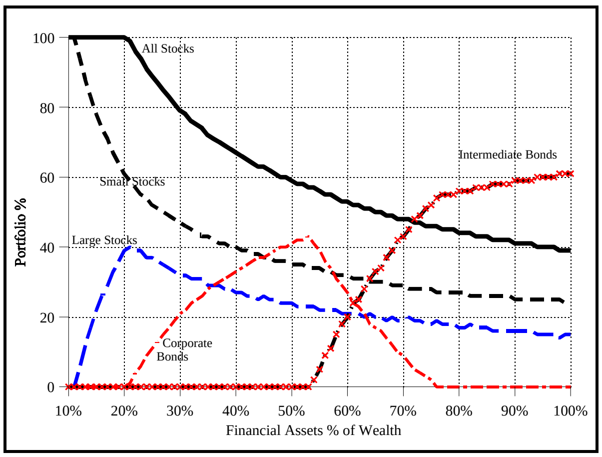
The remainder of this article will focus on the 5 year horizon, as it may be relevant to many households before retirement, for goals with investment horizons of at least 5 years. Even for those investing in retirement accounts might find a 5 year horizon appropriate unless they were certain that they would not need funds before retirement.

Illustration of Results for Five Year Investment Horizon
If a household's wealth is only in financial assets (a very unlikely possibility) the optimal portfolio will depend only on its relative risk aversion. At a relative risk aversion level of 1, all of portfolio should be in stocks: 92% in small stocks and 8% in large stocks. With a relative risk aversion level of 2, half should be in small stocks, 27% in large stocks, and 23% in corporate bonds. The optimal portion for corporate bonds peaks with a relative risk aversion level of 3, then decreases to zero for a relative risk aversion level of 5. For a relative risk aversion level of 6, perhaps typical of Americans based on the Barsky, et al. (1997) results, government intermediate bonds should be 61% of the portfolio, small stocks should be 24%, and large stocks would be 15%.

Adding the optimal portfolio shares for small stocks and large stocks, the optimal proportion for all stocks falls from 100% for a relative risk aversion level of 1 to 30% for a relative risk aversion level of 10. Government long bonds and Treasury bills should not be included for any level of relative risk aversion.

A household's wealth includes human wealth and real estate as well as financial assets. Ideally, human wealth and non-financial assets should be analyzed as parts of a portfolio. However, reliable estimates of the distribution of human wealth rates of return are not available. For simplicity, it is assumed that all non-financial assets maintain their real value during a 5 year period. This assumption is a limitation of the analysis. However, it would be very difficult to create reliable long term estimates of real returns of other major asset categories. For a tenured professor, human wealth slowly decreases as the individual ages, and is unlikely to be subject to large unexpected fluctuations. For those in other occupations, and for households for whom real estate or other non-financial assets are important, the analysis that follows should be interpreted with caution, at least for the results for financial assets representing less than 70% of the portfolio.

Figure 1
Optimal Portfolios, for 5 Year Investment Horizon, By Financial Assets as Percent of Total Wealth, for Relative Risk Aversion = 6 (Moderate Risk Tolerance).

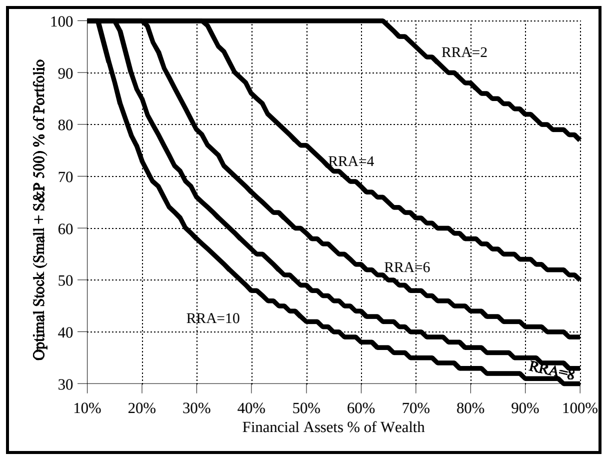


With a relative risk aversion level of 2, the optimal portfolio will be 100% in stocks if financial assets represent no more than 66% of total wealth. All of the portfolio should be in small stocks if financial assets represent no more than 40% of wealth, whereas at 60% of wealth, small stocks should be 71% and large stocks should be 29% of the financial portfolio.

With a relative risk aversion level of 6, the optimal portfolio will be 100% in stocks if financial assets represent no more than 21% of total wealth (Figure 1). All of the portfolio should be in small stocks if financial assets represent no more than 11% of wealth, whereas at 20% of wealth, small stocks should be 62% and large stocks should be 38% of the financial portfolio. If 22% to 77% of wealth is in financial assets, corporate bonds should also be included in the financial portfolio.

With a relative risk aversion level of 10, the optimal portfolio is 100% stocks if financial assets represent no more than 12% of total wealth, 60% in small stocks and 40% in large stocks. If 20% of wealth is in financial assets, corporate bonds should be 26% of the portfolio with the rest in stocks (43% in small stocks and 31% in large stocks). If 30% of wealth is in financial assets, corporate bonds should be 42% of the portfolio with the rest in stocks. However, if financial assets represent 70% of wealth, government intermediate bonds should be 65% of the portfolio.

Figure 2
Optimal Stock Holdings as Percent of Portfolio, for 5 Year Investment Horizon, By Relative Risk Aversion and By Financial Assets as Percent of Total Wealth



If small stocks and large stocks shares are lumped together, the effect of financial assets as a proportion of total wealth is clearer (Figure 2). At all levels of relative risk aversion up to 10, 100% of the portfolio should be in stocks if only 12% of wealth is in financial assets. As the financial asset proportion of wealth increases, the optimal proportion of stocks decreases. However, for a risk aversion level of 6, 100% of an investment portfolio should be in stocks if the portfolio represents 20% of the household's wealth. Even at a relative risk aversion level of 10, stocks should represent 74% of the portfolio.

Conclusions

Summary

This research was based on the assumption that the return from six Ibbotson Associates (1996) asset categories in any future investment horizon (one, 5 or 20 years) is a random selection from all investment horizons of the same length between 1926 and 1995, drawn as sets of returns. The expected utility of all possible portfolios (in increments of 1% for each asset category) was calculated. The portfolio with the highest expected utility for a particular combination of relative risk aversion and level of financial assets as a percent of total wealth was recorded as the optimal portfolio. Stocks, including small stocks, belong in every portfolio. For a 20 year investment horizon, a 100% small stock portfolio dominates every other possible portfolio of Ibbotson assets for a wide range of risk aversion levels.

For 5 and one year investment horizons, even someone who has no human wealth, real estate or other non-financial asset should have substantial holdings of small stocks. For a 5 year investment horizon, corporate and intermediate government bonds should supplement stocks for investors with financial assets representing more than 20% of total wealth and average or above average risk aversion. Based on a 5 year investment horizon, no optimal portfolio contains government long bonds or Treasury bills. The patterns of a one year investment horizon are similar to the patterns of a 5 year horizon, but slightly more conservative, and include small proportions of long term government bonds at high levels of risk aversion and in the rare situation of having financial assets represent almost all of total wealth.

It may be reasonable to define a concept of objective risk tolerance, and have the investment horizon and the ratio of financial assets to total wealth (including human wealth) as the criteria. If the investment horizon is greater than 20 years, and/or the financial asset to wealth ratio is less than 20%, a household should be considered to have high objective risk tolerance. If this ratio is more than 20%, a household should be considered to have low objective risk tolerance for goals with horizons under 20 years. Subjective risk tolerance could be related to the answers to hypothetical questions such as proposed by Barsky, et al. (1997). It is plausible that an individual's subjective risk tolerance will not change with age, whereas objective risk tolerance may increase as one ages.

Implications for Future Research

One limitation of this research is that an individual's wealth was not considered as a portfolio that includes financial assets, human wealth, real estate and other assets, but rather as the sum of the financial portfolio and all other assets, which were treated as not changing unexpectedly. Clearly some explicit treatment of relationships between changes in values of financial assets and other asset categories such as human wealth would be desirable. The empirical estimates of the ratio between financial assets and total wealth (Lee, 1995; Lee & Hanna, 1995) were based on the assumption that real wages would not increase in the future, and therefore were rather conservative for young workers.

The trend away from defined benefit pension plans toward defined contribution plans (e.g., Kennickell, Starr-McCluer & Sundén, 1997) will change the composition of total household wealth. Those workers who will still have a good defined benefit pension plan or who have moderate income, and therefore a relatively high replacement rate from Social Security, may continue to have relatively low ratios of financial assets to total wealth. However, above average income households with no defined benefit pension may have high ratios of financial assets to total wealth as retirement approaches. Households with human wealth tied to financial markets, such as stockbrokers, are in a situation very different (c.f., Jagannathan & Kocherlakota, 1996) from the scenarios discussed in this article

This research did not explicitly consider the possibility of shortfalls in income which could cause problems in covering essential expenses for a household. The expected utility approach theoretically takes this problem into account. However, the 5 year time frame results reported in this article is most appropriate for the typical household before retirement. The one year time frame partially reported in this article may be appropriate for many retired households for whom investment income is not needed for month-to-month necessities. However, for a focus on upper income retirement households, an expected utility analysis based on monthly returns may be appropriate.

Implications for Financial Planning

For a typical investor under the age of 50, a retirement savings portfolio should be 100% in stocks. The advice that the percent in fixed income assets should equal one's age does not seem to be valid even for investors with low subjective risk tolerance until after retirement.

It may be possible to estimate subjective risk tolerance by hypothetical questions such as those used by Barsky, et al. (1997). A modified version may be found on the web (Hanna, 1998). Objective risk tolerance should be based on the investment horizon and the ratio of financial assets to wealth, including human wealth. The household's access to funds for emergencies and other short term goals should also be considered. If a household has no emergency funds, its objective risk tolerance might be considered very low. On the other hand, if a household's primary investment goal is at least 20 years away, then it has very high objective risk tolerance for that goal. In that case, its subjective risk tolerance should not matter. With a 20 year horizon, a household should invest aggressively regardless of its subjective risk tolerance. Additional analysis is needed to further specify optimal investment strategies based on combinations of objective and subjective risk tolerance.[g]

Endnotes

a. *As Siegel (1994) showed, stocks have done well for a long time in the United States, since the early 1800s. Siegel and Thaler (1997) add to the extensive literature on the equity premium puzzle by suggesting alternate explanations for the historically high return to equities compared to bonds, including the possibility that investors do not behave according to economic models of rationality. Given that it is plausible that some combination of institutional restrictions, household liquidity constraints, and individual ignorance and irrationality have contributed to the equity premium puzzle, the expected utility model presented in this article seems plausible to us for normative applications.*

b. *Individuals may appear to be risk neutral for decisions that put a small fraction of their wealth at risk. For instance, even a very risk averse individual might rationally decide to carry cash to the local grocery store, even if there is a chance that it might be lost or stolen.*

c. *Discussion of the equity premium puzzle (e.g., Siegel & Thaler, 1997) implies high levels of relative risk aversion, if consumers are rational and informed. However, the introspective estimates, as well as behavior in everyday life, do not seem consistent with such high estimates of relative risk aversion.*

d. *Bodie and Merton (1998, p. 128) provide a similar example of decreasing human capital and increasing financial wealth until retirement.*

e. *The method used in this article is nonparametric simulation (Ibbotson Associates, 1997, p. 167) , in contrast to methods such as lognormal projection which assume a particular probability distribution.*

f. *The existence of mean-reversion in stock returns could cast doubt on some of the results presented in this article, at least for the one and 5 year results. The literature on mean reversion is mixed, but there have been a number of empirical studies finding evidence of a rejection of the mean-reversion model, including Malliaropulos and Dimitrios (1996) and Cochran and DeFina (1995).*

*One alternative to the method used in this article is to make projections based on the normal distribution. However, such projections do not match historical records very well for long-term investments. If 5 year returns are calculated by assuming a normal distribution, using annual mean and standard deviation for the past 70 years, there would have been a substantial number of cases where the real 5 year accumulation would be less than $0 for small stocks, meaning that an investor lost more than the original principal. This is impossible. The historical records also indicate a longer right tail than normal distribution. The mean reversion model also underestimate the variance for the 5 year real accumulation. Based on historical records from 1926 to 1995 (Ibbotson Associates, 1996), the variance of 5 year real returns of investing $1 at the beginning of each year is 1.314, which is much higher than the variance estimated by mean reversion model using annual data, 0.1121 * 5 = 0.5606.*

Another criticism of the type of assumption used in this article is that there are not very many non-overlapping periods in the Ibbotson data (January 1, 1926 to December 31, 1995.) Although this objection must be acknowledged as a limitation, everyone who invests for a specific goal such as retirement has a starting point an ending point. Each 20 year period, defined in terms of starting points, is very different from other 20 year periods, for instance. Ultimately, though, we must resort to citing the weakness of the alternative approach, as stated in the preceding paragraph.

A simple comparison of the ratio of the mean annualized return to the mean standard deviation for holding periods of 1 to 10 years suggests that the method described in this article underestimates the risk of small stocks for holding periods under 5 years. Therefore, the anomaly that some of the optimal portfolios for one year investment horizons seem riskier than the corresponding portfolios for 5 year horizons may reflect a limitation of this approach to short horizons. However, the general results of this article and other analyses we have conducted, is reasonably convincing to us that the general dominance of small stocks over large stocks for the 5 and 20 year horizons is due to a true superiority of small stocks in expected utility terms.

g. *A crude attempt at portfolio recommendations based on combinations of objective and subjective risk tolerance factors may be found at the following web page: http://www.hec.ohio-state.edu/hanna/risk/risktable.htm*

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